3

Trigonometric Functions

TOPIC 1 Circular System, Trigonometric Ratios, Domain and Range of Trigonometric Functions, Trigonometric Ratios of Allied Angles

1. For any $\theta \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$ the expression

$3(\sin\theta - \cos\theta)^4 + 6(\sin\theta + \cos\theta)^2 + 4\sin^6\theta$ equals: **[Jan. 9, 2019 (I)]**

(a) $13 - 4\cos^2\theta + 6\sin^2\theta\cos^2\theta$
(b) $13 - 4\cos^6\theta$
(c) $13 - 4\cos^2\theta + 6\cos^4\theta$
(d) $13 - 4\cos^4\theta + 2\sin^2\theta\cos^2\theta$

2. Let $f_k(x) = \frac{1}{k}\left(\sin^k x + \cos^k x\right)$ where $x \in R$ and $k \geq 1$. Then $f_4(x) - f_6(x)$ equals **[2014]**

(a) $\frac{1}{4}$ (b) $\frac{1}{12}$
(c) $\frac{1}{6}$ (d) $\frac{1}{3}$

3. If $2\cos\theta + \sin\theta = 1 \left(\theta \neq \frac{\pi}{2}\right)$, then $7\cos\theta + 6\sin\theta$ is equal to: **[Online April 11, 2014]**

(a) $\frac{1}{2}$ (b) 2
(c) $\frac{11}{2}$ (d) $\frac{46}{5}$

4. The expression $\frac{\tan A}{1 - \cot A} + \frac{\cot A}{1 - \tan A}$ can be written as : **[2013]**

(a) sinA cosA + 1 (b) secA cosecA + 1
(c) tanA + cotA (d) secA + cosecA

5. The value of $\cos 255° + \sin 195°$ is **[Online May 26, 2012]**

(a) $\frac{\sqrt{3}-1}{2\sqrt{2}}$ (b) $\frac{\sqrt{3}-1}{\sqrt{2}}$
(c) $-\left(\frac{\sqrt{3}-1}{\sqrt{2}}\right)$ (d) $\frac{\sqrt{3}+1}{\sqrt{2}}$

6. Let $f(x) = \sin x$, $g(x) = x$.

Statement 1: $f(x) \leq g(x)$ for x in $(0, \infty)$

Statement 2: $f(x) \leq 1$ for x in $(0, \infty)$ but $g(x) \to \infty$ as $x \to \infty$. **[Online May 7, 2012]**

(a) Statement 1 is true, Statement 2 is false.
(b) Statement 1 is true, Statement 2 is true, Statement 2 is a correct explanation for Statement 1.
(c) Statement 1 is true, Statement 2 is true, Statement 2 is not a correct explanation for Statement 1.
(d) Statement 1 is false, Statement 2 is true.

7. A triangular park is enclosed on two sides by a fence and on the third side by a straight river bank. The two sides having fence are of same length x. The maximum area enclosed by the park is **[2006]**

(a) $\frac{3}{2}x^2$ (b) $\sqrt{\frac{x^3}{8}}$
(c) $\frac{1}{2}x^2$ (d) πx^2

TOPIC 2 Trigonometric Identities, Conditional Trigonometric Identities, Greatest and Least Value of Trigonometric Expressions

8. The value of $\cos^3\left(\frac{\pi}{8}\right).\cos\left(\frac{3\pi}{8}\right)+\sin^3\left(\frac{\pi}{8}\right).\sin\left(\frac{3\pi}{8}\right)$ is **[Jan. 9, 2020 (I)]**

(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{1}{2\sqrt{2}}$

(c) $\frac{1}{2}$ (d) $\frac{1}{4}$

9. If $\frac{\sqrt{2}\sin\alpha}{\sqrt{1+\cos 2\alpha}}=\frac{1}{7}$ and $\sqrt{\frac{1-\cos 2\beta}{2}}=\frac{1}{\sqrt{10}}$, $\alpha,\beta\in\left(0,\frac{\pi}{2}\right)$, then $\tan(\alpha+2\beta)$ is equal to _____. **[Jan. 8, 2020 (II)]**

10. If $L=\sin^2\left(\frac{\pi}{16}\right)-\sin^2\left(\frac{\pi}{8}\right)$ and $M=\cos^2\left(\frac{\pi}{16}\right)-\sin^2\left(\frac{\pi}{8}\right)$, then : **[Sep. 05, 2020 (II)]**

(a) $L=-\frac{1}{2\sqrt{2}}+\frac{1}{2}\cos\frac{\pi}{8}$ (b) $L=\frac{1}{4\sqrt{2}}-\frac{1}{4}\cos\frac{\pi}{8}$

(c) $M=\frac{1}{4\sqrt{2}}+\frac{1}{4}\cos\frac{\pi}{8}$ (d) $M=\frac{1}{2\sqrt{2}}+\frac{1}{2}\cos\frac{\pi}{8}$

11. The set of all possible values of θ in the interval $(0, \pi)$ for which the points $(1, 2)$ and $(\sin\theta, \cos\theta)$ lie on the same side of the line $x + y = 1$ is : **[Sep. 02, 2020 (II)]**

(a) $\left(0,\frac{\pi}{2}\right)$ (b) $\left(\frac{\pi}{4},\frac{3\pi}{4}\right)$

(c) $\left(0,\frac{3\pi}{4}\right)$ (d) $\left(0,\frac{\pi}{4}\right)$

12. The angle of elevation of the top of a vertical tower standing on a horizontal plane is observed to be 45° from a point A on the plane. Let B be the point 30 m vertically above the point A. If the angle of elevation of the top of the tower from B be 30°, then the distance (in m) of the foot of the tower from the point A is: **[April 12, 2019 (II)]**

(a) $15(3+\sqrt{3})$ (b) $15(5-\sqrt{3})$

13. The value of $\cos^2 10° - \cos 10° \cos 50° + \cos^2 50°$ is : **[April 9, 2019 (II)]**

(a) $\frac{3}{4}+\cos 20°$ (b) 3/4

(c) $\frac{3}{2}(1+\cos 20°)$ (d) 3/2

14. Two poles standing on a horizontal ground are of heights 5m and 10m respectively. The line joining their tops makes an angle of 15° with the ground. Then the distance (in m) between the poles, is: **[April. 09, 2019 (II)]**

(a) $5(2+\sqrt{3})$ (b) $5(\sqrt{3}+1)$

(c) $\frac{5}{2}(2+\sqrt{3})$ (d) $10(\sqrt{3}-1)$

15. The value of $\sin 10° \sin 30° \sin 50° \sin 70°$ is: **[April. 09, 2019 (II)]**

(a) $\frac{1}{16}$ (b) $\frac{1}{32}$

(c) $\frac{1}{18}$ (d) $\frac{1}{36}$

16. If $\cos(\alpha+\beta)=\frac{3}{5}$, $\sin(\alpha-\beta)=\frac{5}{13}$ and $0<\alpha,\beta<\frac{\pi}{4}$, then $\tan(2\alpha)$ is equal to : **[April 8, 2019 (I)]**

(a) $\frac{63}{52}$ (b) $\frac{63}{16}$

(c) $\frac{21}{16}$ (d) $\frac{33}{52}$

17. If $\sin^4\alpha+4\cos^4\beta+2=4\sqrt{2}\sin\alpha\cos\beta$; $\alpha,\beta\in[0,\pi]$, then $\cos(\alpha+\beta)-\cos(\alpha-\beta)$ is equal to : **[Jan. 12, 2019 (II)]**

(a) 0 (b) -1

(c) $\sqrt{2}$ (d) $-\sqrt{2}$

18. Let $f_k(x)=\frac{1}{k}\left(\sin^k x+\cos^k x\right)$ for k = 1, 2, 3, ... Then for all $x\in R$, the value of $f_4(x)-f_6(x)$ is equal to : **[Jan. 11, 2019 (I)]**

(a) $\frac{1}{12}$ (b) $\frac{1}{4}$

(c) $\frac{-1}{12}$ (d) $\frac{5}{12}$

19. The value of **[Jan. 10, 2019 (II)]**

$\cos\frac{\pi}{2^2}\cdot\cos\frac{\pi}{2^3}\cdot\ldots\cdot\cos\frac{\pi}{2^{10}}\cdot\sin\frac{\pi}{2^{10}}$

(a) $\frac{1}{512}$ (b) $\frac{1}{1024}$

20. If $5(\tan^2 x - \cos^2 x) = 2\cos 2x + 9$, then the value of $\cos 4x$ is : **[2017]**

(a) $-\frac{7}{9}$ (b) $-\frac{3}{5}$

(c) $\frac{1}{3}$ (d) $\frac{2}{9}$

21. If m and M are the minimum and the maximum values of $4+\frac{1}{2}\sin^2 2x - 2\cos^4 x, x \in R$, then M – m is equal to : **[Online April 9, 2016]**

(a) $\frac{9}{4}$ (b) $\frac{15}{4}$

(c) $\frac{7}{4}$ (d) $\frac{1}{4}$

22. If $\cos\alpha + \cos\beta = \frac{3}{2}$ and $\sin\alpha + \sin\beta$ $\frac{1}{2}$ and θ is the the arithmetic mean of α and β, then $\sin 2\theta + \cos 2\theta$ is equal to : **[Online April 11, 2015]**

(a) $\frac{3}{5}$ (b) $\frac{7}{5}$

(c) $\frac{4}{5}$ (d) $\frac{8}{5}$

23. If $\operatorname{cosec}\theta = \frac{p+q}{p-q}$ $(p \ne q \ne 0)$, then $\left|\cot\left(\frac{\pi}{4}+\frac{\theta}{2}\right)\right|$ is equal to: **[Online April 9, 2014]**

(a) $\sqrt{\frac{p}{q}}$ (b) $\sqrt{\frac{q}{p}}$

(c) $\sqrt{pq}$ (d) pq

24. If $A = \sin^2 x + \cos^4 x$, then for all real x : **[2011]**

(a) $\frac{13}{16} \le A \le 1$ (b) $1 \le A \le 2$

(c) $\frac{3}{4} \le A \le \frac{13}{16}$ (d) $\frac{3}{4} \le A \le 1$

25. Let $\cos(\alpha+\beta) = \frac{4}{5}$ and $\sin(\alpha-\beta) = \frac{5}{13}$, where $0 \le \alpha, \beta \le \frac{\pi}{4}$. Then $\tan 2\alpha =$ **[2010]**

(a) $\frac{56}{33}$ (b) $\frac{19}{12}$

(c) $\frac{20}{7}$ (d) $\frac{25}{16}$

26. Let **A** and **B** denote the statements

A : $\cos\alpha + \cos\beta + \cos\gamma = 0$

B : $\sin\alpha + \sin\beta + \sin\gamma = 0$

If $\cos(\beta-\gamma) + \cos(\gamma-\alpha) + \cos(\alpha-\beta) = -\frac{3}{2}$, then : **[2009]**

(a) **A** is false and **B** is true

(b) both **A** and **B** are true

(c) both **A** and **B** are false

(d) **A** is true and **B** is false

27. If p and q are positive real numbers such that $p^2 + q^2 = 1$, then the maximum value of $(p+q)$ is **[2007]**

(a) $\frac{1}{2}$ (b) $\frac{1}{\sqrt{2}}$

(c) $\sqrt{2}$ (d) 2.

28. If $0 < x < \pi$ and $\cos x + \sin x = \frac{1}{2}$, then $\tan x$ is **[2006]**

(a) $\frac{(1-\sqrt{7})}{4}$ (b) $\frac{(4-\sqrt{7})}{3}$

(c) $-\frac{(4+\sqrt{7})}{3}$ (d) $\frac{(1+\sqrt{7})}{4}$

29. If $u = \sqrt{a^2\cos^2\theta + b^2\sin^2\theta} + \sqrt{a^2\sin^2\theta + b^2\cos^2\theta}$ then the difference between the maximum and minimum values of u^2 is given by **[2004]**

(a) $(a-b)^2$ (b) $2\sqrt{a^2+b^2}$

(c) $(a+b)^2$ (d) $2(a^2+b^2)$

30. Let α, β be such that $\pi < \alpha - \beta < 3\pi$.

If $\sin\alpha + \sin\beta = -\frac{21}{65}$ and $\cos\alpha + \cos\beta = -\frac{27}{65}$, then the value of $\cos\frac{\alpha-\beta}{2}$ **[2004]**

(a) $\frac{-6}{65}$ (b) $\frac{3}{\sqrt{130}}$

(c) $\frac{6}{65}$ (d) $-\frac{3}{\sqrt{130}}$

31. The function $f(x) = \log\left(x + \sqrt{x^2+1}\right)$, is **[2003]**

(a) neither an even nor an odd function

(b) an even function

(c) an odd function

(d) a periodic function.

32. The period of $\sin^2\theta$ is **[2002]**

(a) π^2 (b) π

33. Which one is not periodic? **[2002]**

(a) $|\sin 3x| + \sin^2 x$ (b) $\cos\sqrt{x} + \cos^2 x$

(c) $\cos 4x + \tan^2 x$ (d) $\cos 2x + \sin x$

TOPIC 3 Solutions of Trigonometric Equations

34. If the equation $\cos^4\theta + \sin^4\theta + \lambda = 0$ has real solutions for θ, then λ lies in the interval : **[Sep. 02, 2020 (II)]**

(a) $\left(-\frac{5}{4}, -1\right)$ (b) $\left[-1, -\frac{1}{2}\right]$

(c) $\left(-\frac{1}{2}, -\frac{1}{4}\right]$ (d) $\left[-\frac{3}{2}, -\frac{5}{4}\right]$

35. The number of distinct solutions of the equation,

$\log_{1/2}|\sin x| = 2 - \log_{1/2}|\cos x|$ in the interval $[0, 2\pi]$, is _____. **[Jan. 9, 2020 (I)]**

36. The number of solutions of the equation

$1 + \sin^4 x = \cos^2 3x, x \in \left[-\frac{5\pi}{2}, \frac{5\pi}{2}\right]$ is : **[April 12, 2019 (I)]**

(a) 3 (b) 5

(c) 7 (d) 4

37. Let S be the set of all $\alpha \in R$ such that the equation, $\cos 2x + \alpha \sin x = 2\alpha - 7$ has a solution. Then S is equal to : **[April 12, 2019 (II)]**

(a) R (b) [1, 4]

(c) [3,7] (d) [2, 6]

38. If $[x]$ denotes the greatest integer $\le x$, then the system of linear equations

$[\sin\theta]\, x + [-\cos\theta]\, y = 0$

$[\cot\theta]\, x + y = 0$ **[April 12, 2019 (II)]**

(a) have infinitely many solutions if $\theta \in \left(\frac{\pi}{2}, \frac{2\pi}{3}\right)$ and has a unique solution if $\theta \in \left(\pi, \frac{7\pi}{6}\right)$.

(b) has a unique solution if $\theta \in \left(\frac{\pi}{2}, \frac{2\pi}{3}\right) \cup \left(\pi, \frac{7\pi}{6}\right)$.

(c) has a unique solution if $\theta \in \left(\frac{\pi}{2}, \frac{2\pi}{3}\right)$ and have infinitely many solutions if $\theta \in \left(\pi, \frac{7\pi}{6}\right)$.

(d) have infinitely many solutions if $\theta \in \left(\frac{\pi}{2}, \frac{2\pi}{3}\right) \cup \left(\pi, \frac{7\pi}{6}\right)$

39. Let $S = \{\theta \in [-2\pi, 2\pi] : 2\cos^2\theta + 3\sin\theta = 0\}$. Then the sum of the elements of S is: **[April 9, 2019 (I)]**

(a) $\frac{13\pi}{6}$ (b) $\frac{5\pi}{3}$

(c) 2π (d) π

40. If $0 \le x < \frac{\pi}{2}$, then the number of values of x for which $\sin x - \sin 2x + \sin 3x = 0$, is: **[Jan. 09, 2019 (II)]**

(a) 3 (b) 1

(c) 4 (d) 2

41. The number of solutions of $\sin 3x = \cos 2x$, in the interval $\left(\frac{\pi}{2}, \pi\right)$ is **[Online April 15, 2018]**

(a) 3 (b) 4

(c) 2 (d) 1

42. If sum of all the solutions of the equation $8\cos x \cdot \left(\cos\left(\frac{\pi}{6}+x\right)\cdot\cos\left(\frac{\pi}{6}-x\right) - \frac{1}{2}\right) - 1$ in $[0, \pi]$ is $k\pi$, then k is equal to : **[2018]**

(a) $\frac{13}{9}$ (b) $\frac{8}{9}$

(c) $\frac{20}{9}$ (d) $\frac{2}{3}$

43. If $0 \le x < 2\pi$, then the number of real values of x, which satisfy the equation $\cos x + \cos 2x + \cos 3x + \cos 4x = 0$ is: **[2016]**

(a) 7 (b) 9

(c) 3 (d) 5

44. The number of $x \in [0, 2\pi]$ for which

$\left|\sqrt{2\sin^4 x + 18\cos^2 x} - \sqrt{2\cos^4 x + 18\sin^2 x}\right| = 1$ is **[Online April 9, 2016]**

(a) 2 (b) 6

(c) 4 (d) 8

45. The number of values of α in $[0, 2\pi]$ for which $2\sin^3\alpha - 7\sin^2\alpha + 7\sin\alpha = 2$, is: **[Online April 9, 2014]**

(a) 6 (b) 4

(c) 3 (d) 1

46. Let $A = \{\theta : \sin(\theta) = \tan(\theta)\}$ and $B = \{\theta : \cos(\theta) = 1\}$ be two sets. Then : **[Online April 25, 2013]**

(a) A = B

(b) $A \not\subset B$

(c) $B \not\subset A$

(d) $A \subset B$ and $B - A \ne \phi$

47. The number of solutions of the equation $\sin 2x - 2\cos x + 4\sin x = 4$ in the interval $[0, 5\pi]$ is : **[Online April 23, 2013]**

(a) 3 (b) 5
(c) 4 (d) 6

48. **Statement-1:** The number of common solutions of the trigonometric equations $2\sin^2\theta - \cos 2\theta = 0$ and $2\cos^2\theta - 3\sin\theta = 0$ in the interval $[0, 2\pi]$ is two.

Statement-2: The number of solutions of the equation, $2\cos^2\theta - 3\sin\theta = 0$ in the interval $[0, \pi]$ is two.

[Online April 22, 2013]

(a) Statement-1 is true; Statement-2 is true; Statement-2 is a correct explanation for statement-1.
(b) Statement-1 is true; Statement-2 is true; Statement-2 is **not** a correct explanation for statement-1.
(c) Statement-1 is false; Statement-2 is true.
(d) Statement-1 is true; Statement-2 is false.

49. The equation $e^{\sin x} - e^{-\sin x} - 4 = 0$ has : **[2012]**

(a) infinite number of real roots
(b) no real roots
(c) exactly one real root
(d) exactly four real roots

50. The possible values of $\theta \in (0, \pi)$ such that $\sin(\theta) + \sin(4\theta) + \sin(7\theta) = 0$ are **[2011RS]**

(a) $\frac{\pi}{4}, \frac{5\pi}{12}, \frac{\pi}{2}, \frac{2\pi}{3}, \frac{3\pi}{4}, \frac{8\pi}{9}$
(b) $\frac{2\pi}{9}, \frac{\pi}{4}, \frac{\pi}{2}, \frac{2\pi}{3}, \frac{3\pi}{4}, \frac{35\pi}{36}$
(c) $\frac{2\pi}{9}, \frac{\pi}{4}, \frac{\pi}{2}, \frac{2\pi}{3}, \frac{3\pi}{4}, \frac{8\pi}{9}$
(d) $\frac{2\pi}{9}, \frac{\pi}{4}, \frac{4\pi}{9}, \frac{\pi}{2}, \frac{3\pi}{4}, \frac{8\pi}{9}$

51. The number of values of x in the interval $[0, 3\pi]$ satisfying the equation $2\sin^2 x + 5\sin x - 3 = 0$ is **[2006]**

(a) 4 (b) 6
(c) 1 (d) 2

52. The number of solution of $\tan x + \sec x = 2\cos x$ in $[0, 2\pi)$ is **[2002]**

(a) 2 (b) 3
(c) 0 (d) 1

Hints & Solutions

1. **(b)** $3(\sin\theta-\cos\theta)^4+6(\sin\theta+\cos\theta)^2+4\sin^6\theta$

$=3(1-2\sin\theta\cos\theta)^2+6(1+2\sin\theta\cos\theta)+4\sin^6\theta$

$=3(1+4\sin^2\theta\cos^2\theta-4\sin\theta\cos\theta)+6-12\sin\theta\cos\theta+4\sin^6\theta$

$=9+12\sin^2\theta\cos^2\theta+4\sin^6\theta$

$=9+12\cos^2\theta(1-\cos^2\theta)+4(1-\cos^2\theta)^3$

$=9+12\cos^2\theta-12\cos^4\theta+4(1-\cos^6\theta-3\cos^2\theta+3\cos^4\theta)$

$=9+4-4\cos^6\theta$

$=13-4\cos^6\theta$

2. **(b)** Let $f_k(x)=\frac{1}{k}(\sin^k x+\cos^k x)$

Consider $f_4(x)-f_6(x)=\frac{1}{4}(\sin^4 x+\cos^4 x)-\frac{1}{6}(\sin^6 x+\cos^6 x)$

$=\frac{1}{4}[1-2\sin^2 x\cos^2 x]-\frac{1}{6}[1-3\sin^2 x\cos^2 x]$

$=\frac{1}{4}-\frac{1}{6}=\frac{1}{12}$

3. **(d)** Given $2\cos\theta+\sin\theta=1$

Squaring both sides, we get

$(2\cos\theta+\sin\theta)^2=1^2$

$\Rightarrow 4\cos^2\theta+\sin^2\theta+4\sin\theta\cos\theta=1$

$\Rightarrow 3\cos^2\theta+(\cos^2\theta+\sin^2\theta)+4\sin\theta\cos\theta=1$

$\Rightarrow 3\cos^2\theta+\cancel{1}+4\sin\theta\cos\theta=\cancel{1}$

$\Rightarrow 3\cos^2\theta+4\sin\theta\cos\theta=0$

$\Rightarrow \cos\theta(3\cos\theta+4\sin\theta)=0$

$\Rightarrow 3\cos\theta+4\sin\theta=0\Rightarrow 3\cos\theta=-4\sin\theta$

$\Rightarrow \frac{-3}{4}=\tan\theta=\sqrt{\sec^2\theta-1}=\frac{-3}{4}$

$\left(\because \tan\theta=\sqrt{\sec^2\theta-1}\right)$

$\Rightarrow \sec^2\theta-1=\left(\frac{-3}{4}\right)^2=\frac{9}{16}$

$\Rightarrow \sec^2\theta=\frac{9}{16}+1=\frac{25}{16}\Rightarrow\sec\theta=\frac{5}{4}$

or $\boxed{\cos\theta=\frac{4}{5}}$ ...(1)

Now, $\sin^2\theta+\cos^2\theta=1\Rightarrow\sin^2\theta+\left(\frac{4}{5}\right)^2=1$

$\sin^2\theta+\frac{4}{5}=1\Rightarrow\sin^2\theta=1-\frac{16}{25}=\frac{9}{25}$

$\sin\theta=\pm\frac{3}{5}$ ...(2)

Taking $\left(\sin\theta=+\frac{3}{5}\right)$ because $\left(\sin\theta=-\frac{3}{5}\right)$ cannot satisfy the given equation.

Therefore; $7\cos\theta+6\sin\theta$

$=7\times\frac{4}{5}+6\times\frac{3}{5}=\frac{28}{5}+\frac{18}{5}=\frac{46}{5}$

4. **(b)** Given expression can be written as

$\frac{\sin A}{\cos A}\times\frac{\sin A}{\sin A-\cos A}+\frac{\cos A}{\sin A}\times\frac{\cos A}{\cos A-\sin A}$

$\left(\because \tan A=\frac{\sin A}{\cos A}\text{ and } \cot A=\frac{\cos A}{\sin A}\right)$

$=\frac{1}{\sin A-\cos A}\left\{\frac{\sin^3 A-\cos^3 A}{\cos A\sin A}\right\}$

$\because a^3-b^3=(a-b)(a^2+ab+b^2)$

$=\frac{\sin^2 A+\sin A\cos A+\cos^2 A}{\sin A\cos A}$

= 1 + sec A cosec A

5. **(c)** Consider $\cos 255^\circ+\sin 195^\circ$

$=\cos(270^\circ-15^\circ)+\sin(180^\circ+15^\circ)$

$=-\sin 15^\circ-\sin 15^\circ$

$=-2\sin 15^\circ=-2\left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right)=-\left(\frac{\sqrt{3}-1}{\sqrt{2}}\right)$

6. **(c)** Let $f(x)=\sin x$ and $g(x)=x$

Statement-1: $f(x)\le g(x)\ \forall x\in(0,\infty)$

i.e., $\sin x\le x\ \forall x\in(0,\infty)$

which is true

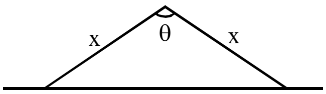
Statement-2: $f(x)\le 1\ \forall\ x\in(0,\infty)$

i.e., $\sin x\le 1\ \forall\ x\in(0,\infty)$

It is true and

$g(x)=x\to\infty$ as $x\to\infty$ also true.

7. **(c)** Area $= \frac{1}{2}x^2 \sin\theta$



Maximum value of $\sin\theta$ is 1 at $\theta = \frac{\pi}{2}$

$A_{max} = \frac{1}{2}x^2$

8. **(b)** $\cos^3\frac{\pi}{8}\left[4\cos^3\frac{\pi}{8} - 3\cos\frac{\pi}{8}\right] + \sin^3\frac{\pi}{8}\left[3\sin\frac{\pi}{8} - 4\sin^3\frac{\pi}{8}\right]$

$= 4\cos^6\frac{\pi}{8} - 4\sin^6\frac{\pi}{8} - 3\cos^4\frac{\pi}{8} + 3\sin^4\frac{\pi}{8}$

$= 4\left[\left(\cos^2\frac{\pi}{8} - \sin^2\frac{\pi}{8}\right)\right]$

$\left[\left(\sin^4\frac{\pi}{8} + \cos^4\frac{\pi}{8} + \sin^2\frac{\pi}{8}\cos^2\frac{\pi}{8}\right)\right]$

$-3\left[\left(\cos^2\frac{\pi}{8} - \sin^2\frac{\pi}{8}\right)\left(\cos^2\frac{\pi}{8} + \sin^2\frac{\pi}{8}\right)\right]$

$= \cos\frac{\pi}{4}\left[4\left(1 - \sin^2\frac{\pi}{8}\cos^2\frac{\pi}{8}\right) - 3\right]$

$= \frac{1}{\sqrt{2}}\left[1 - \frac{1}{2}\right] = \frac{1}{2\sqrt{2}}$

9. **(1)** $\frac{\sqrt{2}\sin\alpha}{\sqrt{2}\cos\alpha} = \frac{1}{7}$ and $\sqrt{\frac{1-\cos^2\beta}{2}} = \frac{1}{10}$

$\Rightarrow \frac{\sqrt{2}\sin\beta}{\sqrt{2}} = \frac{1}{\sqrt{10}}$

$\therefore \tan\alpha = \frac{1}{7}$ and $\sin\beta = \frac{1}{\sqrt{10}}$

$\tan\beta = \frac{1}{3}$

$\therefore \tan 2\beta = \frac{2\tan\beta}{1-\tan^2\beta} = \frac{2.\frac{1}{3}}{1-\frac{1}{9}} = \frac{\frac{2}{3}}{\frac{8}{9}} = \frac{3}{4}$

$\tan(\alpha + 2\beta) = \frac{\tan\alpha + \tan 2\beta}{1 - \tan\alpha\tan 2\beta}$

$= \frac{\frac{1}{7} + \frac{3}{4}}{1 - \frac{1}{7}.\frac{3}{4}} = \frac{\frac{4+21}{28}}{\frac{25}{28}} = 1$

10. **(d)** $L + M = 1 - 2\sin^2\frac{\pi}{8} = \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$ ...(i)

and $L - M = -\cos\frac{\pi}{8}$ ...(ii)

From equations (i) and (ii),

$L = \frac{1}{2}\left(\frac{1}{\sqrt{2}} - \cos\frac{\pi}{8}\right) = \frac{1}{2\sqrt{2}} - \frac{1}{2}\cos\frac{\pi}{8}$ and

$M = \frac{1}{2}\left(\frac{1}{\sqrt{2}} + \cos\frac{\pi}{8}\right) = \frac{1}{2\sqrt{2}} + \frac{1}{2}\cos\frac{\pi}{8}$

11. **(a)** Let $f(x, y) = x + y - 1$

Given $(1, 2)$ and $(\sin\theta, \cos\theta)$ are lies on same side.

$\therefore f(1, 2)\cdot f(\sin\theta, \cos\theta) > 0$

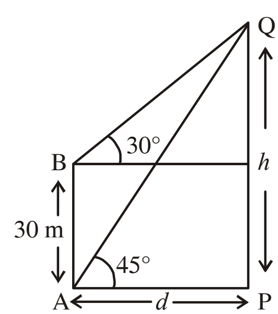
$\Rightarrow 2[\sin\theta + \cos\theta - 1] > 0$

$\Rightarrow \sin\theta + \cos\theta > 1 \Rightarrow \sin\left(\theta + \frac{\pi}{4}\right) > \frac{1}{\sqrt{2}}$

$\Rightarrow \theta + \frac{\pi}{4} \in \left(\frac{\pi}{4}, \frac{3\pi}{4}\right) \Rightarrow \theta \in \left(0, \frac{\pi}{2}\right)$

12. **(a)** Let the height of the tower be h and distance of the foot of the tower from the point A is d.

By the diagram,



$\tan 45° = \frac{h}{d} = 1$

$h = d$ ...(i)

$\tan 30° = \frac{h-30}{d}$

$\sqrt{3}(h-30) = d$ ...(ii)

Put the value of h from (i) to (ii),

$\sqrt{3}d = d + 30\sqrt{3}$

$d = \frac{30\sqrt{3}}{\sqrt{3}-1} = 15\sqrt{3}\left(\sqrt{3}+1\right) = 15\left(3+\sqrt{3}\right)$

13. (b) $\cos^2 10° - \cos 10° \cos 50° + \cos^2 50°$

$$= \left(\frac{1+\cos 20°}{2}\right) + \left(\frac{1+\cos 100°}{2}\right) - \frac{1}{2}(2\cos 10° \cos 50°)$$

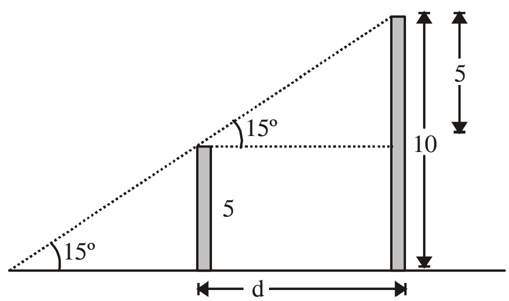
$$= 1 + \frac{1}{2}(\cos 20° + \cos 100°) - \frac{1}{2}[\cos 60° + \cos 40°]$$

$$= \left(1 - \frac{1}{4}\right) + \frac{1}{2}[\cos 20° + \cos 100° - \cos 40°]$$

$$= \frac{3}{4} + \frac{1}{2}[2\cos 60° \times \cos 40° - \cos 40°]$$

$$= \frac{3}{4}$$

14. (a)



By the diagram,

$$\tan 15° = \frac{5}{d} \Rightarrow d = \frac{5}{\tan 15°} = \frac{5(\sqrt{3}+1)}{\sqrt{3}-1}$$

$$= \frac{5(4+2\sqrt{3})}{2} = 5(2+\sqrt{3})$$

15. (a) $\because \sin(60° + A).\sin(60° - A) \sin A = \frac{1}{4} \sin 3A$

$\therefore \sin 10° \sin 50° \sin 70° = \sin 10° \sin(60° - 10°)$

$$\sin(60° + 10°) = \frac{1}{4}\sin 30°$$

$$\Rightarrow \sin 10° \sin 30° \sin 50° \sin 70° = \frac{1}{4}\sin^2 30 = \frac{1}{16}$$

16. (b) $\because \alpha + \beta$ and $\alpha - \beta$ both are acute angles.

$$\cos(\alpha+\beta) = \frac{3}{5}, \text{ then } \sin(\alpha+\beta) = \sqrt{1-\left(\frac{3}{5}\right)^2} = \frac{4}{5}$$

$$\tan(\alpha+\beta) = \frac{4}{3}$$

And $\sin(\alpha - \beta) = \frac{5}{13}$, then

$$\cos(\alpha-\beta) = \sqrt{1-\left(\frac{5}{13}\right)^2} = \frac{12}{13}$$

$$\Rightarrow \tan(\alpha - \beta) = \frac{5}{12}$$

Now, $\tan 2\alpha = \tan((\alpha+\beta)+(\alpha-\beta))$

$$= \frac{\tan(\alpha+\beta)+\tan(\alpha-\beta)}{1-\tan(\alpha+\beta).\tan(\alpha-\beta)} = \frac{\frac{4}{3}+\frac{5}{12}}{1-\frac{4}{3}.\frac{5}{12}} = \frac{63}{16}$$

17. (d) $\because$ The given equation is

$\sin^4\alpha + 4\cos^4\beta + 2 = 4\sqrt{2}\sin\alpha \cdot \cos\beta,\ \alpha, \beta \in [0, \pi]$

Then, by A.M., G.M. ineqality;

A.M. ≥ G.M.

$$\frac{\sin^4\alpha + 4\cos^4\beta + 1 + 1}{4} \geq \left(\sin^4\alpha \cdot 4\cos^4\beta \cdot 1 \cdot 1\right)^{\frac{1}{4}}$$

$\sin^4\alpha + 4\cos^4\beta + 1 + 1 \geq 4\sqrt{2}\sin\alpha \cdot |\cos\beta|$

Inequality still holds when $\cos\beta < 0$ but L.H.S. is positive than $\cos\beta > 0$, then

L.H.S. = R.H.S

$$\therefore \quad \sin^4\alpha = 1 \text{ and } \cos^4\beta = \frac{1}{4}$$

$$\Rightarrow \quad \alpha = \frac{\pi}{2} \text{ and } \beta = \frac{\pi}{4}$$

$\therefore \quad \cos(\alpha+\beta) - \cos(\alpha-\beta)$

$$= \cos\left(\frac{\pi}{2}+\beta\right) - \cos\left(\frac{\pi}{2}-\beta\right)$$

$$= -\sin\beta - \sin\beta = -2\sin\frac{\pi}{4} = -\sqrt{2}$$

18. (a) $f_k(x) = \frac{1}{k}(\sin^k x + \cos^k x)$

$$f_4(x) = \frac{1}{4}[\sin^4 x + \cos^4 x]$$

$$= \frac{1}{4}\left[(\sin^2 x + \cos^2 x)^2 - \frac{(\sin 2x)^2}{2}\right]$$

$$= \frac{1}{4}\left[1 - \frac{(\sin 2x)^2}{2}\right]$$

$$f_6(x) = \frac{1}{6}[\sin^6 x + \cos^6 x]$$

$$= \frac{1}{6}\left[(\sin^2 x + (\cos^2 x) - \frac{3}{4}(\sin^2 x)^2\right]$$

$=\frac{1}{6}\left[1-\frac{3}{4}(\sin 2x)^2\right]$

Now $f_4(x)-f_{(6)}(x)=\frac{1}{4}-\frac{1}{6}-\frac{(\sin 2x)^2}{8}+\frac{1}{8}(\sin 2x)^2$

$=\frac{1}{12}$

19. (a) $A=\cos\frac{\pi}{2^2}.\cos\frac{\pi}{2^3}\ldots\cos\frac{\pi}{2^{10}}.\sin\frac{\pi}{2^{10}}$

$=\frac{1}{2}\left(\cos\frac{\pi}{2^2}.\cos\frac{\pi}{2^3}\ldots\cos\frac{\pi}{2^9}\sin\frac{\pi}{2^9}\right)$

$=\frac{1}{2^8}\left(\cos\frac{\pi}{2^2}.\sin\frac{\pi}{2^2}\right)=\frac{1}{2^9}\sin\frac{\pi}{2}$

$=\frac{1}{512}$

20. (a) We have

$5\tan^2 x-5\cos^2 x=2(2\cos^2 x-1)+9$

$\Rightarrow 5\tan^2 x-5\cos^2 x=4\cos^2 x-2+9$

$\Rightarrow 5\tan^2 x=9\cos^2 x+7$

$\Rightarrow 5(\sec^2 x-1)=9\cos^2 x+7$

Let $\cos^2 x=t$

$\Rightarrow \frac{5}{t}-9t-12=0$

$\Rightarrow 9t^2+12t-5=0$

$\Rightarrow 9t^2+15t-3t-5=0$

$\Rightarrow (3t-1)(3t+5)=0$

$\Rightarrow t=\frac{1}{3}$ as $t\neq-\frac{5}{3}$.

$\cos 2x=2\cos^2 x-1=2\left(\frac{1}{3}\right)-1=-\frac{1}{3}$

$\cos 4x=2\cos^2 2x-1=2\left(-\frac{1}{3}\right)^2-1=-\frac{7}{9}$

21. (b) $4+\frac{1}{2}\sin^2 2x-2\cos^4 x$

$4+2(1-\cos^2 x)\cos^2 x-2\cos^4 x$

$-4\left\{\cos^4 x-\frac{\cos^2 x}{2}-1+\frac{1}{16}-\frac{1}{16}\right\}$

$-4\left\{\left(\cos^2 x-\frac{1}{4}\right)^2-\frac{17}{16}\right\}$

$0\le\cos^2 x\le 1$

$-\frac{1}{4}\le\cos^2 x-\frac{1}{4}\le\frac{3}{4}$

$0\le\left(\cos^2 x-\frac{1}{4}\right)^2\le\frac{9}{16}$

$-\frac{17}{16}\le\left(\cos^2 x-\frac{1}{4}\right)^2-\frac{17}{16}\le\frac{9}{16}-\frac{17}{16}$

$\frac{17}{4}\ge-4\left\{\left(\cos^2 x-\frac{1}{4}\right)^2-\frac{17}{16}\right\}\ge\frac{1}{2}$

$M=\frac{17}{4}$

$m=\frac{1}{2}$

$M-m=\frac{17}{4}-\frac{2}{4}=\frac{15}{4}$

22. (b) Let $\cos\alpha+\cos\beta=\frac{3}{2}$

$\Rightarrow 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}=\frac{3}{2}$ ...(i)

and $\sin\alpha+\sin\beta=\frac{1}{2}$

$\Rightarrow 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}=\frac{1}{2}$ ...(ii)

On dividing (ii) by (i), we get

$\tan\left(\frac{\alpha+\beta}{2}\right)=\frac{1}{3}$

Given : $\theta=\frac{\alpha+\beta}{2}\Rightarrow 2\theta=\alpha+\beta$

Consider $\sin 2\theta+\cos 2\theta=\sin(\alpha+\beta)+\cos(\alpha+\beta)$

$=\frac{\frac{2}{3}}{1+\frac{1}{9}}+\frac{1-\frac{1}{9}}{1+\frac{1}{9}}=\frac{6}{10}+\frac{8}{10}=\frac{7}{5}$

23. (b) $\text{cosec}\theta=\frac{p+q}{p-q}$, $\sin\theta=\frac{p-q}{p+q}$

$\cos\theta=\pm\sqrt{1-\sin^2\theta}=\sqrt{1-\left(\frac{p-q}{p+q}\right)^2}=\frac{2\sqrt{pq}}{(p+q)}$

$\left|\cot\left(\frac{\pi}{4}+\frac{\theta}{2}\right)\right|=\left|\frac{\cot\frac{\pi}{4}\cot\frac{\theta}{2}-1}{\cot\frac{\pi}{4}+\cot\frac{\theta}{2}}\right|=\left|\frac{\cot\frac{\theta}{2}-1}{\cot\frac{\theta}{2}+1}\right|$

$=\left|\frac{\cos\frac{\theta}{2}-\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}+\sin\frac{\theta}{2}}\right|$

On rationalizing denominator, we get

$$\left|\left(\frac{\cos\frac{\theta}{2}-\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}+\sin\frac{\theta}{2}}\right)\left(\frac{\cos\frac{\theta}{2}+\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}+\sin\frac{\theta}{2}}\right)\right|$$

$$=\left|\frac{\cos\theta}{\sin^2\frac{\theta}{2}+\cos^2\frac{\theta}{2}+2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}\right|$$

$$=\left|\frac{\cos\theta}{1+\sin\theta}\right|=\left|\frac{2\sqrt{pq}/(p+q)}{1+\frac{(p-q)}{p+q}}\right|=\frac{\sqrt{pq}}{p}=\sqrt{\frac{q}{p}}$$

24. (d) $A=\sin^2 x+\cos^4 x$

$=\sin^2 x+\cos^2 x(1-\sin^2 x)$

$=\sin^2 x+\cos^2 x-\frac{1}{4}(2\sin x.\cos x)^2$

$=1-\frac{1}{4}\sin^2(2x)$

$\because -1\le \sin 2x\le 1$

$\Rightarrow\ 0\le \sin^2(2x)\le 1$

$\Rightarrow\ 0\ge -\frac{1}{4}\sin^2(2x)\ge -\frac{1}{4}$

$\Rightarrow 1\ge 1-\frac{1}{4}\sin^2(2x)\ge 1-\frac{1}{4}$

$\Rightarrow 1\ge A\ge \frac{3}{4}$

25. (a) $\cos(\alpha+\beta)=\frac{4}{5}\ \Rightarrow \tan(\alpha+\beta)=\frac{3}{4}$

$\sin(\alpha-\beta)=\frac{5}{13}\Rightarrow \tan(\alpha-\beta)=\frac{5}{12}$

$\tan 2\alpha=\tan\left[(\alpha+\beta)+(\alpha-\beta)\right]$

$$=\frac{\tan(\alpha+\beta)+\tan(\alpha-\beta)}{1-\tan(\alpha+\beta)\tan(\alpha-\beta)}=\frac{\frac{3}{4}+\frac{5}{12}}{1-\frac{3}{4}.\frac{5}{12}}=\frac{56}{33}$$

26. (b) Given that

$\cos(\beta-\gamma)+\cos(\gamma-\alpha)+\cos(\alpha-\beta)=-\frac{3}{2}$

$\Rightarrow\ 2[\cos(\beta-\gamma)+\cos(\gamma-\alpha)+\cos(\alpha-\beta)]+3=0$

$\Rightarrow\ 2[\cos(\beta-\gamma)+\cos(\gamma-\alpha)+\cos(\alpha-\beta)] + \sin^2\alpha+\cos^2\alpha+\sin^2\beta+\cos^2\beta + \sin^2\gamma+\cos^2\alpha=0$

$\Rightarrow\ [\sin^2\alpha+\sin^2\beta+\sin^2\gamma+2\sin\alpha\ \sin\beta + 2\sin\beta\sin\gamma+2\sin\gamma\sin\alpha]+[\cos^2\alpha+\cos^2\beta + \cos^2\gamma+2\cos\alpha\cos\beta+2\cos\beta\cos\gamma + 2\cos\gamma\ \cos\alpha]=0$

$\Rightarrow\ [\sin\alpha+\sin\beta+\sin\gamma]^2+(\cos\alpha+\cos\beta+\cos\gamma)^2=0$

$\Rightarrow\ \sin\alpha+\sin\beta+\sin\gamma=0$ and $\cos\alpha+\cos\beta+\cos\gamma=0$

$\therefore$ A and B both are true.

27. (c) Given that $p^2+q^2=1$

$\therefore\ p=\cos\theta$ and $q=\sin\theta$ satisfy the given equation

Then $p+q=\cos\theta+\sin\theta$

We know that

$-\sqrt{a^2+b^2}\le a\cos\theta+b\sin\theta\le\sqrt{a^2+b^2}$

$\therefore -\sqrt{2}\le\cos\theta+\sin\theta\le\sqrt{2}$

Hence max. value of $p+q$ is $\sqrt{2}$

28. (c) $\cos x+\sin x=\frac{1}{2}\ \Rightarrow\ 1+\sin 2x=\frac{1}{4}$

$\Rightarrow\ \sin 2x=-\frac{3}{4}$,

$\therefore \pi<2x<2\pi$

$\Rightarrow\ \frac{\pi}{2}<x\le\pi$ ...(i)

$\frac{2\tan x}{1+\tan^2 x}=-\frac{3}{4}$

$\Rightarrow\ 3\tan^2 x+8\tan x+3=0$

$\therefore\ \tan x=\frac{-8\pm\sqrt{64-36}}{6}=-\frac{-4\pm\sqrt{7}}{3}$

for $\frac{\pi}{2}<x<\pi$, $\tan x<0$

$\therefore\ \tan x=\frac{-4-\sqrt{7}}{3}$

29. (a) $u^2=a^2+b^2+2\sqrt{\begin{matrix}(a^4+b^4)\cos^2\theta\sin^2\theta\\ +a^2b^2(\cos^4\theta+\sin^4\theta)\end{matrix}}$ ...(1)

Now, $(a^4+b^4)\cos^2\theta\sin^2\theta+a^2b^2(\cos^4\theta+\sin^4\theta)$

$=(a^4+b^4)\cos^2\theta\sin^2\theta+a^2b^2(1-2\cos^2\theta\sin^2\theta)$

$=(a^4+b^4-2a^2b^2)\cos^2\theta\sin^2\theta+a^2b^2$

$=(a^2-b^2)^2.\frac{\sin^2 2\theta}{4}+a^2b^2$ ...(2)

$\because 0\le\sin^2 2\theta\le 1$

$\Rightarrow 0\le(a^2-b^2)^2\frac{\sin^2 2\theta}{4}\le\frac{(a^2-b^2)^2}{4}$

$\Rightarrow a^2b^2\le(a^2-b^2)^2\frac{\sin^2 2\theta}{4}+a^2b^2$

$\le(a^2-b^2)^2.\frac{1}{4}+a^2b^2$ ....(3)

From (1)

$$a^2+b^2+2\sqrt{a^2b^2} \le u^2 \le a^2+b^2+\frac{2}{2}\sqrt{\left(a^2+b^2\right)^2}$$

$$(a+b)^2 \le u^2 \le 2\left(a^2+b^2\right)$$

$\therefore$ Max. value – Min. value

$$= 2(a^2+b^2)-(a+b^2)=(a-b)^2$$

30. **(d)** $\pi < \alpha-\beta < 3\pi$

$$\Rightarrow \frac{\pi}{2} < \frac{\alpha-\beta}{2} < \frac{3\pi}{2} \Rightarrow \cos\frac{\alpha-\beta}{2} < 0 \quad ....(1)$$

$$\sin\alpha+\sin\beta = -\frac{21}{65}$$

$$\Rightarrow 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2} = -\frac{21}{65} \quad ....(2)$$

$$\cos\alpha+\cos\beta = -\frac{27}{65}$$

$$\Rightarrow 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2} = -\frac{27}{65} \quad ....(3)$$

Squaring and adding (2) and (3), we get

$$4\cos^2\frac{\alpha-\beta}{2} = \frac{(21)^2+(27)^2}{(65)^2} = \frac{1170}{65\times 65}$$

$$\therefore \cos^2\frac{\alpha-\beta}{2} = \frac{9}{130} \Rightarrow \cos\frac{\alpha-\beta}{2} = -\frac{3}{\sqrt{130}} \quad \text{[from (1)]}$$

31. **(c)** Given $f(x)=\log(x+\sqrt{x^2+1})$

$$f(-x)=\log\left\{-x+\sqrt{x^2+1}\right\} = \log\left\{\frac{x^2-x^2+1}{x+\sqrt{x^2+1}}\right\}$$

$$= -\log(x+\sqrt{x^2+1}) = -f(x)$$

$\Rightarrow f(x)$ is an odd function.

32. **(b)** We know that $\sin^2\theta = \dfrac{1-\cos 2\theta}{2}$;

Since period of $\cos 2\theta = \dfrac{2\pi}{2} = \pi$

Hence period of $\sin^2\theta$ is also π.

33. **(b)** we know that $\cos\sqrt{x}$ is non periodic

$\therefore$ $\cos\sqrt{x}+\cos^2 x$ can not be periodic.

34. **(b)** $\sin^4\theta+\cos^4\theta = -\lambda$

$$\Rightarrow (\sin^2\theta+\cos^2\theta)^2 - 2\sin^2\theta\cdot\cos^2\theta = -\lambda$$

$$\Rightarrow 1-2\sin^2\theta\cos^2\theta = -\lambda$$

$$\Rightarrow \lambda = \frac{(\sin 2\theta)^2}{2} - 1$$

$\Rightarrow$ as $\sin^2 2\theta \in [0, 1]$

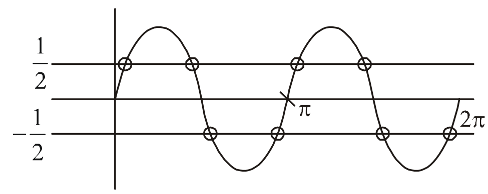
$$\Rightarrow \lambda \in \left[-1, \frac{-1}{2}\right]$$

35. **(8)** $\log_{1/2}|\sin x| = 2-\log_{1/2}|\cos x|$

$\Rightarrow$ $\log_{1/2}|\sin x\cos x| = 2$

$\Rightarrow$ $|\sin x\cos x| = \dfrac{1}{4}$

$\Rightarrow$ $\sin 2x = \pm\dfrac{1}{2}$



Hence, total number of solutions = 8.

36. **(c)** Consider equation, $1+\sin^4 x = \cos^2 3x$

L.H.S. $= 1+\sin^4 x$ and R.H.S. $= \cos^2 3x$

$\because$ L.H.S. ≥ 1 and R.H.S. $\le 1 \Rightarrow$ L.H.S. = R.H.S. = 1

$\sin^4 x = 0$, and $\cos^2 3x = 1$

$\Rightarrow$ $\sin x = 0$ and $(4\cos^2 x-3)^2\cos^2 x = 1$

$\Rightarrow$ $\sin x = 0$ and $\cos^2 x = 1 \Rightarrow x = 0, \pm\pi, \pm 2\pi$

Hence, total number of solutions is 5.

37. **(d)** Given equation is, $\cos 2x+\alpha\sin x = 2\alpha-7$

$1-2\sin^2 x+\alpha\sin x = 2\alpha-7$

$2\sin^2 x-\alpha\sin x+(2\alpha-8)=0$

$$\Rightarrow \quad \sin x = \frac{\alpha\pm\sqrt{\alpha^2-8(2\alpha+8)}}{4}$$

$$\Rightarrow \quad \sin x = \frac{\alpha\pm(\alpha-8)}{4} \Rightarrow \sin x = \frac{\alpha-4}{4}$$

[$\sin x = 2$ (rejected)]

$\because$ equation has solution, then $\dfrac{\alpha-4}{4} \in [-1,1]$

$\Rightarrow$ $\alpha \in [2,6]$

38. **(a)** According to the question, there are two cases.

Case 1 : $\theta \in \left(\dfrac{\pi}{2}, \dfrac{2\pi}{3}\right)$

In this interval, $[\sin\theta]=0$, $[-\cos\theta]=0$ and $[\cot\theta]=-1$

Then the system of equations will be ;

$0\cdot x+0\cdot y=0$ and $-x+y=0$

Which have infinitely many solutions.

Case 2 : $\theta \in \left(\pi, \dfrac{7\pi}{6}\right)$

In this interval, $[\sin\theta]=-1$ and $[-\cos\theta]=0$

Then the system of equations will be ;

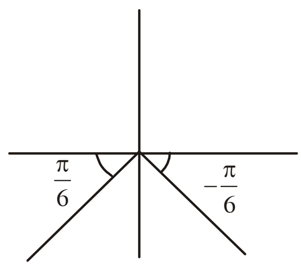
$-x+0\cdot y=0$ and $[\cot\theta]x+y=0$

Clearly, $x=0$ and $y=0$ which has unique solution.

39. **(c)** $2\cos^2\theta+3\sin\theta=0$

$(2\sin\theta+1)(\sin\theta-2)=0$

$\Rightarrow \sin\theta=-\frac{1}{2}$ or $\sin\theta=2\rightarrow$ Not possibe



The required sum of all solutions in $[-2\pi, 2\pi]$ is

$$=\left(\pi+\frac{\pi}{6}\right)+\left(2\pi-\frac{\pi}{6}\right)+\left(-\frac{\pi}{6}\right)+\left(-\pi+\frac{\pi}{6}\right)=2\pi$$

40. **(d)** $\sin x-\sin 2x+\sin 3x=0$

$\Rightarrow \sin x-2\sin x.\cos x+3\sin x-4\sin^3 x=0$

$\Rightarrow 4\sin x-4\sin^3 x-2\sin x.\cos x=0$

$\Rightarrow 2\sin x(1-\sin^2 x)-\sin x.\cos x=0$

$\Rightarrow 2\sin x.\cos^2 x-\sin x.\cos x=0$

$\Rightarrow \sin x.\cos x(2\cos x-1)=0$

$\therefore \sin x=0, \cos x=0, \cos x=\frac{1}{2}$

$\therefore x=0,\frac{\pi}{3} \quad \because x\in\left[0,\frac{\pi}{2}\right)$

41. **(d)** $\sin 3x=\cos 2x$

$\Rightarrow 3\sin x-4\sin^3 x=1-2\sin^2 x$

$\Rightarrow 4\sin^3 x-2\sin^2 x-3\sin x+1=0$

$\Rightarrow \sin x=1, \frac{-2\pm 2\sqrt{5}}{8}$

In the interval $\left(\frac{\pi}{2},\pi\right)$, $\sin x=\frac{-2+2\sqrt{5}}{8}$

So, there is only one solution.

42. **(a)** $\because 8\cos x\left(\cos^2\frac{\pi}{6}-\sin^2 x-\frac{1}{2}\right)=1$

$\Rightarrow 8\cos x\left(\frac{3}{4}-\frac{1}{2}-\sin^2 x\right)=1$

$\Rightarrow 8\cos x\left(\frac{1}{4}-(1-\cos^2 x)\right)=1$

$\Rightarrow 8\cos x\left(\frac{1}{4}-1+\cos^2 x\right)=1$

$\Rightarrow 8\cos x\left(\cos^2 x-\frac{3}{4}\right)=1$

$\Rightarrow 8\left(\frac{4\cos^3 x-3\cos x}{4}\right)=1$

$\Rightarrow 2(4\cos^3 x-3\cos x)=1$

$\Rightarrow 2\cos 3x=1\Rightarrow \cos 3x=\frac{1}{2}$

$\therefore 3x=2n\pi\pm\frac{\pi}{3}, n\in 1$

$\Rightarrow x=\frac{2n\pi}{3}\pm\frac{\pi}{9}$

In $x\in[0,\pi]: x=\frac{\pi}{9},\frac{2\pi}{3}+\frac{\pi}{9},\frac{2\pi}{3}-\frac{\pi}{9}$, only

Sum of all the solutions of the equation

$$=\left(\frac{1}{9}+\frac{2}{3}+\frac{1}{9}+\frac{2}{3}-\frac{1}{9}\right)\pi=\frac{13}{9}\pi$$

43. **(a)** $\cos x+\cos 2x+\cos 3x+\cos 4x=0$

$\Rightarrow 2\cos 2x\cos x+2\cos 3x\cos x=0$

$\Rightarrow 2\cos x\left(2\cos\frac{5x}{2}\cos\frac{x}{2}\right)=0$

$\cos x=0, \cos\frac{5x}{2}=0, \cos\frac{x}{2}=0$

$x=\pi,\frac{\pi}{2},\frac{3\pi}{2},\frac{\pi}{5},\frac{3\pi}{5},\frac{7\pi}{5},\frac{9\pi}{5}$

44. **(d)** $\left|\sqrt{2\sin^4 x+18\cos^2 x}-\sqrt{2\cos^4 x+18\sin^2 x}\right|=1$

$\sqrt{2\sin^4 x+18\cos^2 x}-\sqrt{2\cos^4 x+18\sin^2 x}=\pm 1$

$\sqrt{2\sin^4 x+18\cos^2 x}=\pm 1+\sqrt{2\cos^4 x+18\sin^2 x}$

by squaring both the sides we will get 8 solutions

45. **(c)** $2\sin^3\alpha-7\sin^2\alpha+7\sin\alpha-2=0$

$\Rightarrow 2\sin^2\alpha(\sin\alpha-1)-5\sin\alpha(\sin\alpha-1)+2(\sin\alpha-1)=0$

$\Rightarrow (\sin\alpha-1)(2\sin^2\alpha-5\sin\alpha+2)=0$

$\Rightarrow \sin\alpha-1=0$ or $2\sin^2\alpha-5\sin\alpha+2=0$

$\sin\alpha=1$ or $\sin\alpha=\frac{5\pm\sqrt{25-16}}{4}=\frac{5\pm 3}{4}$

$\alpha=\frac{\pi}{2}$ or $\sin\alpha=\frac{1}{2},2$

Now, $\sin\alpha\neq 2$

for, $\sin \alpha = \frac{1}{2}$

$\alpha = \frac{\pi}{3}, \frac{2\pi}{3}$

There are three values of α between $[0, 2\pi]$

46. **(b)** Let $A = \{\theta : \sin\theta = \tan\theta\}$

and $B = \{\theta : \cos\theta = 1\}$

Now, $A = \left\{\theta : \sin\theta = \frac{\sin\theta}{\cos\theta}\right\}$

$= \{\theta : \sin\theta(\cos\theta - 1) = 0\}$

$= \{\theta = 0, \pi, 2\pi, 3\pi, ...\}$

For $B : \cos\theta = 1 \Rightarrow \theta = \pi, 2\pi, 4\pi, ...$

This shows that A is not contained in B. i.e. $A \not\subset B$. but $B \subset A$.

47. **(a)** $\sin 2x - 2\cos x + 4\sin x = 4$

$\Rightarrow 2\sin x \cdot \cos x - 2\cos x + 4\sin x - 4 = 0$

$\Rightarrow (\sin x - 1)(\cos x - 2) = 0$

$\because \cos x - 2 \neq 0, \therefore \sin x = 1$

$\therefore x = \frac{\pi}{2}, \frac{5\pi}{2}, \frac{9\pi}{2}$

48. **(b)** $2\sin^2\theta - \cos 2\theta = 0$

$\Rightarrow 2\sin^2\theta - (1 - 2\sin^2\theta) = 0$

$\Rightarrow 2\sin^2\theta - 1 + 2\sin^2\theta = 0$

$\Rightarrow 4\sin^2\theta = 1 \Rightarrow \sin\theta = \pm\frac{1}{2}$

$\therefore \theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}, \theta \in [0, 2\pi]$

$\therefore \theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

Now $2\cos^2\theta - 3\sin\theta = 0$

$\Rightarrow 2(1 - \sin^2\theta) - 3\sin\theta = 0$

$\Rightarrow -2\sin^2\theta - 3\sin\theta + 2 = 0$

$\Rightarrow -2\sin^2\theta - 4\sin\theta + \sin\theta + 2 = 0$

$\Rightarrow 2\sin^2\theta - \sin\theta + 4\sin\theta - 2 = 0$

$\Rightarrow \sin\theta(2\sin\theta - 1) + 2(2\sin\theta - 1) = 0$

$\Rightarrow \sin\theta = \frac{1}{2}, -2$

But $\sin\theta = -2$, is not possible

$\therefore \sin\theta = \frac{1}{2}, -2 \Rightarrow \theta = \frac{\pi}{6}, \frac{5\pi}{6}$

Hence, there are two common solution, there each of the statement-1 and 2 are true but statement-2 is not a correct explanation for statement-1.

49. **(b)** Given equation is $e^{\sin x} - e^{-\sin x} - 4 = 0$

Put $e^{\sin x} = t$ in the given equation, we get

$t^2 - 4t - 1 = 0$

$\Rightarrow t = \frac{4 \pm \sqrt{16+4}}{2} = \frac{4 \pm \sqrt{20}}{2}$

$= \frac{4 \pm 2\sqrt{5}}{2} = 2 \pm \sqrt{5}$

$\Rightarrow e^{\sin x} = 2 \pm \sqrt{5}$ $(\because t = e^{\sin x})$

$\Rightarrow e^{\sin x} = 2 - \sqrt{5}$ and $e^{\sin x} = 2 + \sqrt{5}$

$\Rightarrow e^{\sin x} = 2 - \sqrt{5} < 0$

and $\sin x = \ln(2 + \sqrt{5}) > 1$

So, rejected.

Hence, given equation has no solution.

$\therefore$ The equation has no real roots.

50. **(d)** $\sin 4\theta + 2\sin 4\theta \cos 3\theta = 0$

$\sin 4\theta(1 + 2\cos 3\theta) = 0$

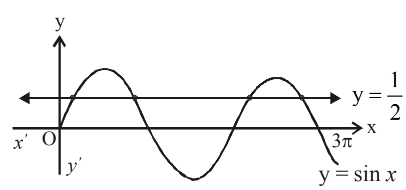
$\sin 4\theta = 0$ or $\cos 3\theta = -\frac{1}{2}$

$4\theta = n\pi; n \in I$

or $3\theta = 2n\pi \pm \frac{2\pi}{3}, n \in I$

$\theta = \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}$ or $\theta = \frac{2\pi}{9}, \frac{8\pi}{9}, \frac{4\pi}{9}$ $[\because \theta, \in (0, \pi)]$

51. **(a)**



$2\sin^2 x + 5\sin x - 3 = 0$

$\Rightarrow (\sin x + 3)(2\sin x - 1) = 0$

$\Rightarrow \sin x = \frac{1}{2}$ and $\sin x \neq -3$

$\therefore$ In $[0, 3\pi]$, x has 4 values.

52. **(b)** $\because \tan x + \sec x = 2\cos x;$

$\Rightarrow \sin x + 1 = 2\cos^2 x$

$\Rightarrow \sin x + 1 = 2(1 - \sin^2 x);$

$\Rightarrow 2\sin^2 x + \sin x - 1 = 0;$

$\Rightarrow (2\sin x - 1)(\sin x + 1) = 0$

$\Rightarrow \sin x = \frac{1}{2}, -1.;$

$\Rightarrow x = 30°, 150°, 270°.$

Number of solution = 3